How Hall electric fields intrinsically chaotize and heat ions during collisionless magnetic reconnection

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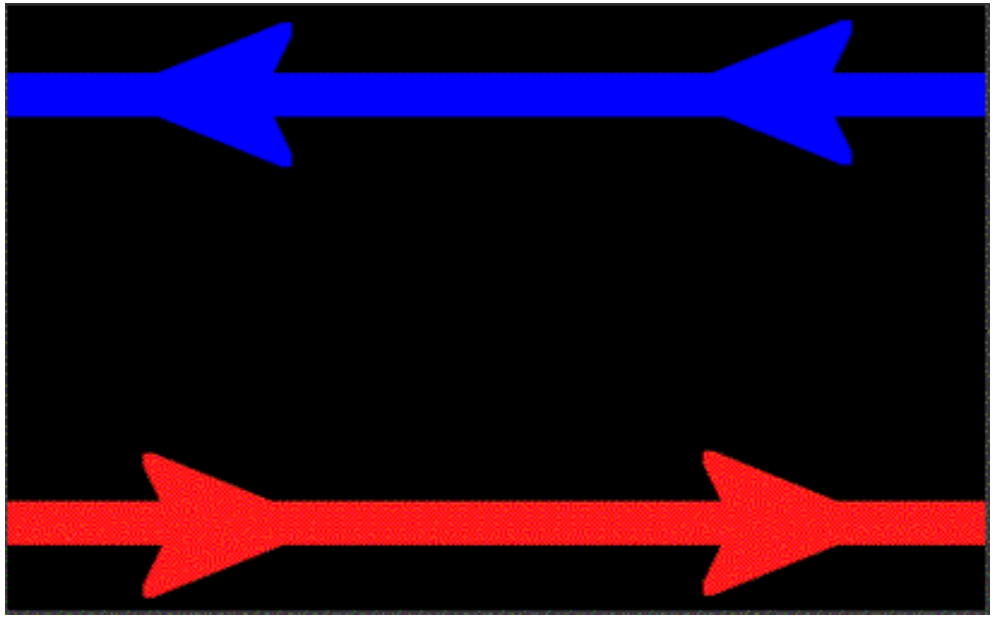
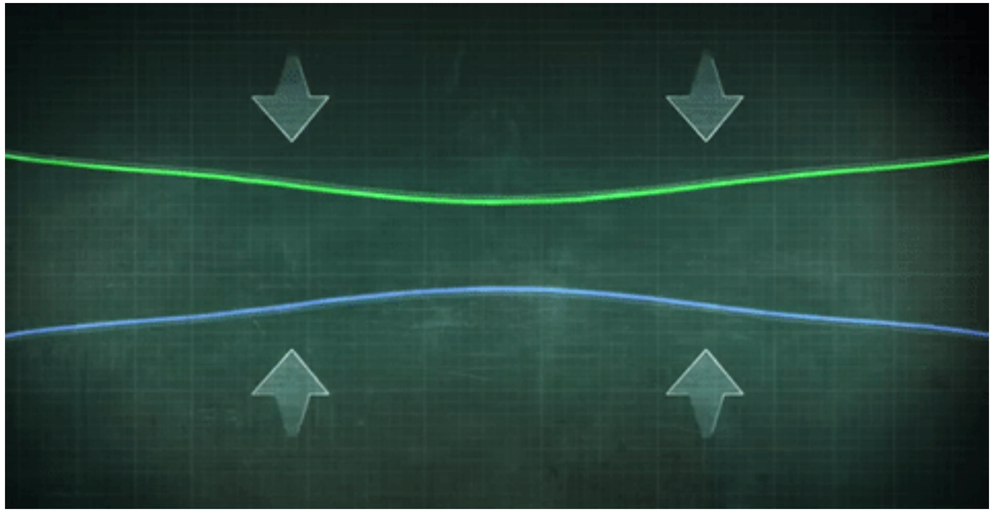
Outline

- Magnetic reconnection and ion heating
- Stochastic heating
- Fluid derivation using canonical vorticity
- Kinetic analysis
- Particle-in-cell simulations
- Conclusion

Magnetic Reconnection





- Fundamental magnetized plasma phenomenon
- Opposing magnetic field lines come together, annihilate, and reconnect
- Common in magnetized plasmas; often abrupt and spontaneous
- Stored magnetic energy => particle kinetic/thermal energy

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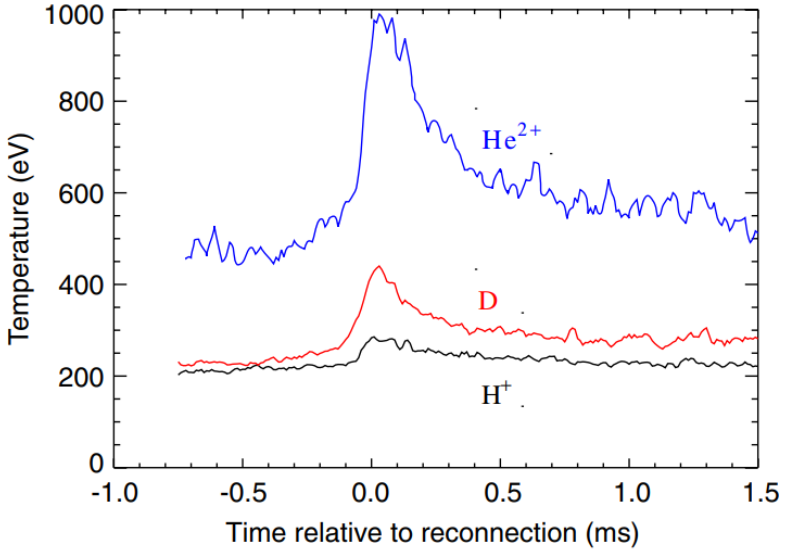
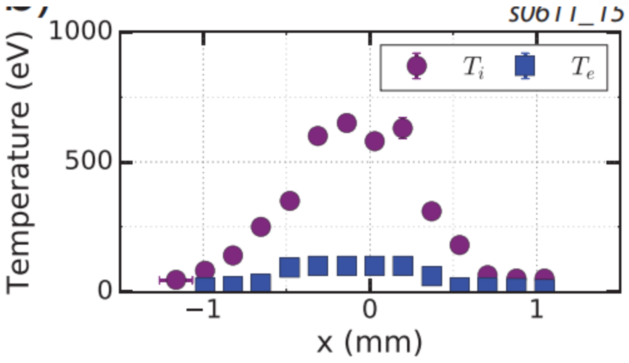
Ion Heating in Reconnection

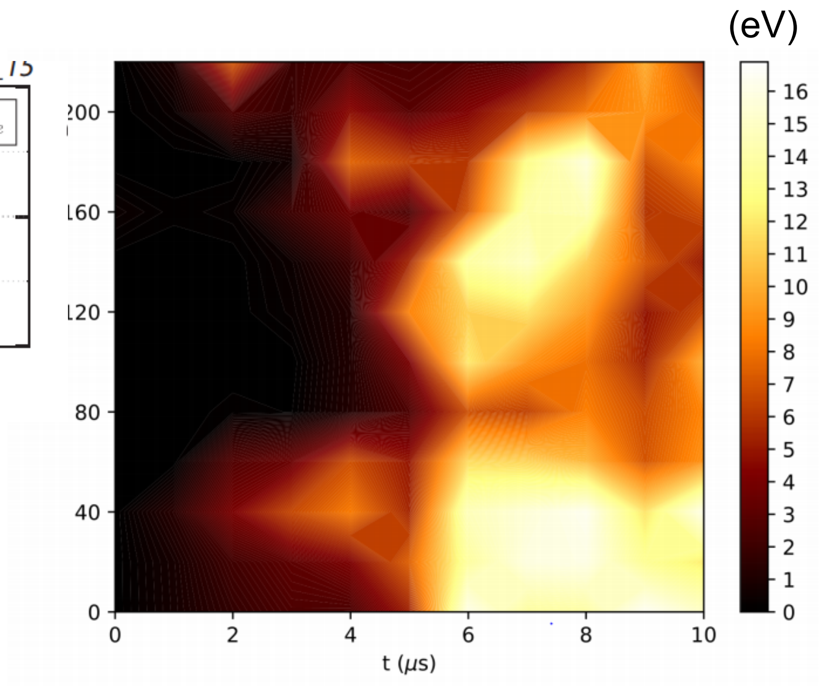
- Fast, strong ion heating is observed with reconnection



Fiksel et al, 2009



Hare et al, 2017



Seo et al., 2020

- Mass-dependent (higher mass => higher temperature)

- Related to coronal heating problem

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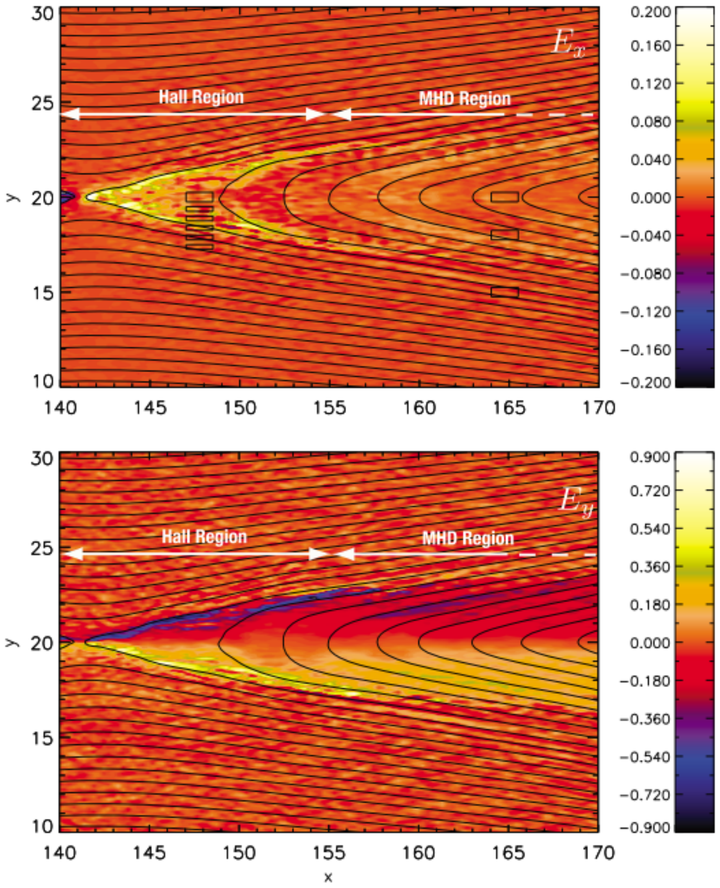
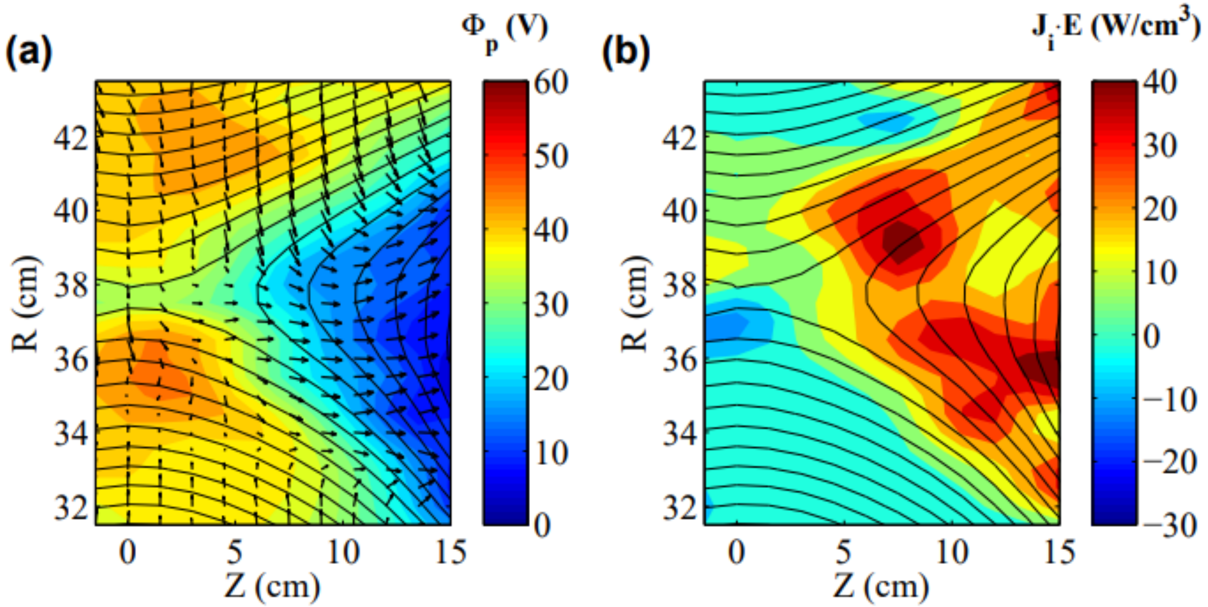
Ion Heating in Reconnection

- In-plane Hall electric fields are attributed to ion energization
- No consensus on the heating mechanism

Simulation (Aunai et al., 2011)



Experiment (Yoo et al., 2013)

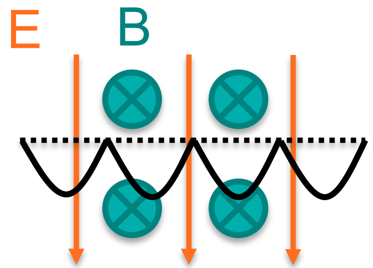


Stochastic Heating

- Occurs when the potential gradient perpendicular to the local magnetic field is large enough

$$\frac{m_i}{q_i B^2}\left|\nabla_\perp^2 \phi\right| > 1$$

- Chaotic motion => significant heating
- Involves breakdown of guiding center motion



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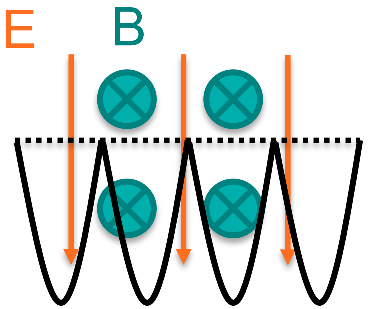

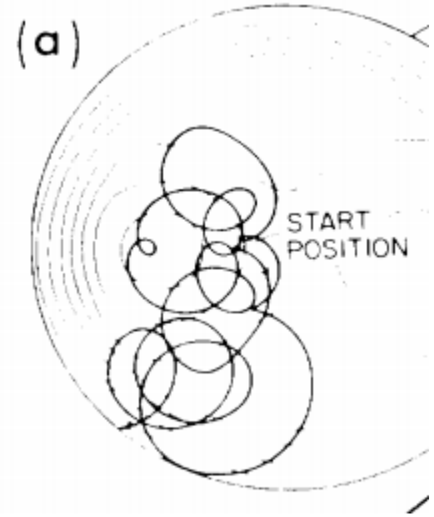

McChesney et al., 1987

- Observed in the solar wind and corona, collisionless shocks, and experiments (Vranjes et al., 2009; 2010; Chandran et al., 2010; McChesney et al., 1987; Stasiewicz et al., 2020)

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Stochastic Heating

- Consider

$$\mathbf{E} = E\hat{y}\sin(ky - \omega t),$$

$$\mathbf{B} = B\hat{z}.$$

- Then,

$$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2} = \hat{x}\frac{E}{B}\sin(ky - \omega t)$$

- and

$$\mathbf{v}_p = \frac{m}{qB^2}\frac{d\mathbf{E}_\perp}{dt},$$

$$= \hat{y}\frac{mE}{qB^2}\frac{d}{dt}\sin(ky - \omega t),$$

$$= \hat{y}\frac{mE}{qB^2}\left(k\overset{\mathbf{v}_p}{\boxed{\frac{dy}{dt}}} - \omega\right)\cos(ky - \omega t).$$

- Solution is

$$\mathbf{v}_p = -\hat{y}\frac{\omega mE}{qB^2}\frac{\cos(ky - \omega t)}{1 - \alpha\cos(ky - \omega t)},$$

where

$$\alpha = \frac{mkE}{qB^2} \Longleftrightarrow \frac{m_i}{q_i B^2}\left|\nabla_\perp^2\phi\right| > 1$$

- When α>1, polarization drift >> ExB drift => nonlinear chaotic motion

- Phase space broadening => (perpendicular) heating

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Canonical Vorticity

$$m_e \frac{D\mathbf{u}_e}{Dt} = q_e\mathbf{E} + q_e\mathbf{u}_e \times \mathbf{B}$$

$$m_e \frac{\partial \mathbf{u}_e}{\partial t} + m_e\mathbf{u}_e \cdot \nabla \mathbf{u}_e = q_e\mathbf{E} + q_e\mathbf{u}_e \times \mathbf{B}$$

Take curl

$$\frac{\partial \mathbf{Q}_e}{\partial t} = \nabla \times (\mathbf{u}_e \times \mathbf{Q}_e)$$

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Canonical Vorticity

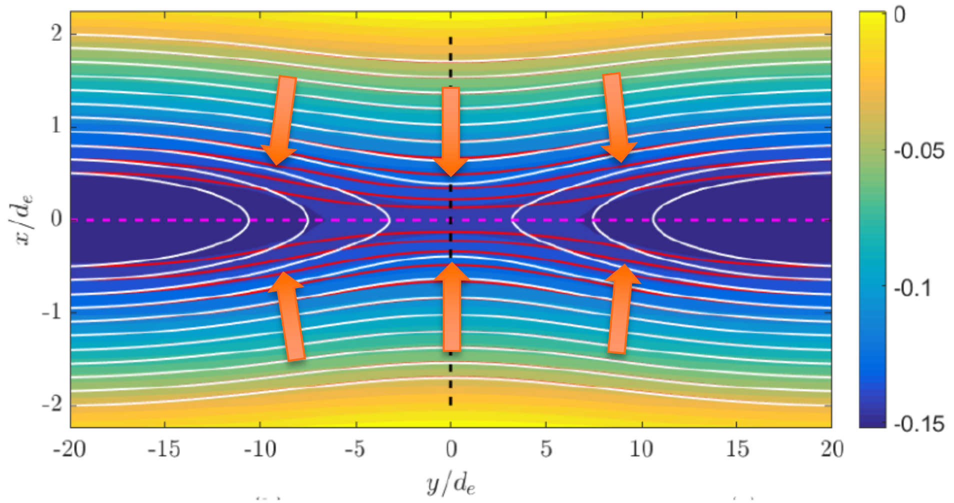
$$\mathbf{Q}_e = m_e \nabla \times \mathbf{u}_e + q_e \mathbf{B}$$
$$= \nabla \times (m_e \mathbf{u}_e + q_e \mathbf{A})$$
$$= \nabla \times \mathbf{P}_e$$

Q is curl of canonical momentum, or **canonical vorticity**

$$\frac{\partial \mathbf{Q}_e}{\partial t} = \nabla \times (\mathbf{u}_e \times \mathbf{Q}_e) \quad \Longleftrightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B})$$

Q is frozen into the electron fluid
Q flux is conserved => **Q cannot reconnect**

In-plane Potential



- White: **B**
- Red: **Q**
- Color: ϕ
- Arrows: **E**

$$q_e\mathbf{E} = m_e\frac{D\mathbf{u}_e}{Dt} - q_e\mathbf{u}_e \times \mathbf{B} + \frac{\nabla p_e}{n_e}$$

$$q_e\mathbf{E} = m_e\frac{\partial \mathbf{u}_e}{\partial t} - \mathbf{u}_e \times \mathbf{Q}_e + \nabla\left(\frac{m_e u_e^2}{2}\right) + \cancel{\frac{\nabla p_e}{n_e}}$$

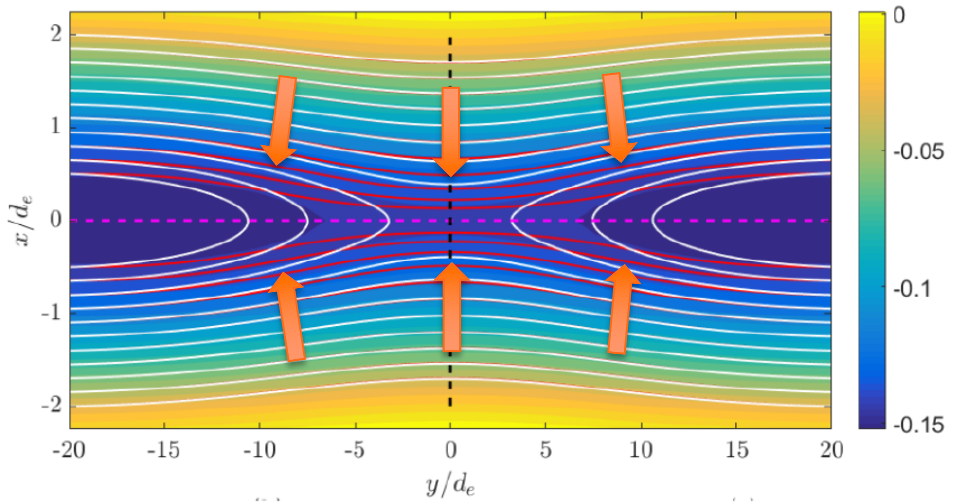
$$q_e E_y = m_e\frac{\partial u_{ey}}{\partial t} \cancel{- u_{ez}Q_{ex}} + u_{ex}Q_{ez} + \frac{\partial}{\partial y}\left(\frac{m_e u_e^2}{2}\right)$$

$$E_y = -\frac{\partial}{\partial y}\left(\frac{m_e\left[u_e^2 + u_{ey}^2\right]}{2e}\right)$$

Similarly,

$$E_x = -\frac{\partial}{\partial x}\left(\frac{B^2}{2\mu_0 n_e e}\right)$$

In-plane Potential



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Stochastic ion heating criterion:

$$\frac{m_i}{q_i B^2}\left|\nabla_\perp^2 \phi\right| > 1 \quad \Longrightarrow$$

- Inflow criterion $L_x^2 < d_i^2$
- Outflow criterion $m_i / m_e > 1$

Stochastic heating is intrinsic to collisionless reconnection!

Kinetic Analysis

- Harris equilibrium magnetic field (exact kinetic equilibrium)

$$B_y(x) = 2\sqrt{\mu_0 n_0 k_B T}\tanh\left(\frac{x}{\lambda}\right),$$

- Associated electric field:

$$E_x(x) = -VB_y(x) = -2V\sqrt{\mu_0 n_0 k_B T}\tanh\left(\frac{x}{\lambda}\right),$$

- Insert into stochastic heating criterion:

$$\frac{m_i}{q_i[B_y^2(x) + B_z^2]}\left|\frac{\partial E_x(x)}{\partial x}\right| > 1,$$

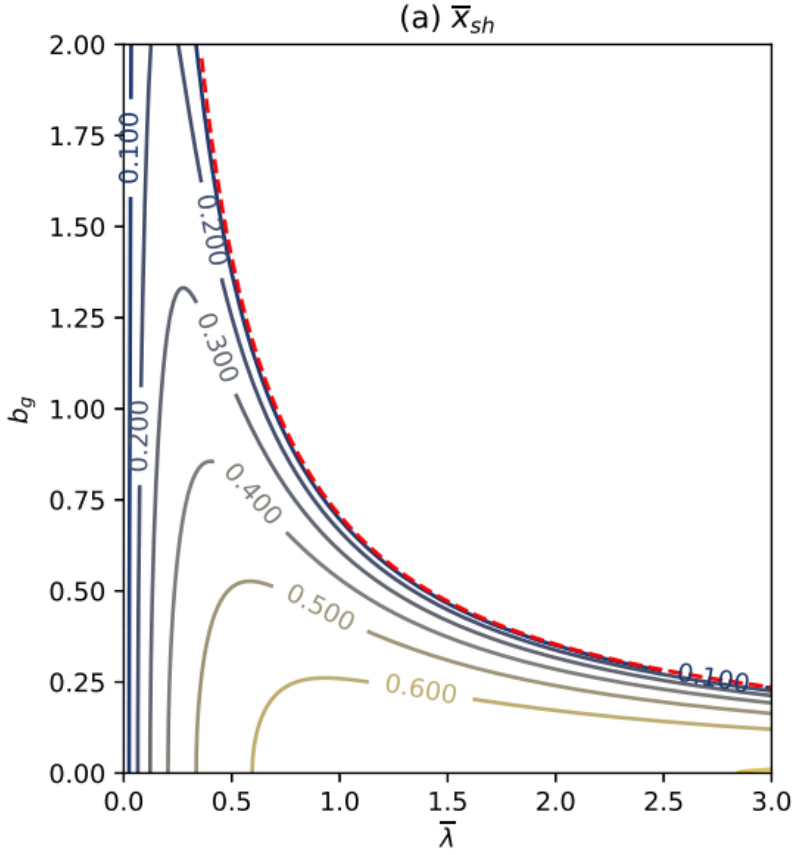
- Equation to solve (b_g is the relative guide field strength)

$$2\frac{\lambda^2}{d_i^2}\cosh^2\left(\frac{x}{\lambda}\right)\left[\tanh^2\left(\frac{x}{\lambda}\right) + b_g^2\right] < 1,$$

Kinetic Analysis

- Solution is (bar means normalized to d_i)

$$\bar{x}_{\rm sh} = \frac{\bar{\lambda}}{2}\ln\frac{\bar{\lambda}^2(1-b_g^2)+1+\sqrt{(1-2\bar{\lambda}^2 b_g^2)(2\bar{\lambda}^2+1)}}{\bar{\lambda}^2(1+b_g^2)}.$$



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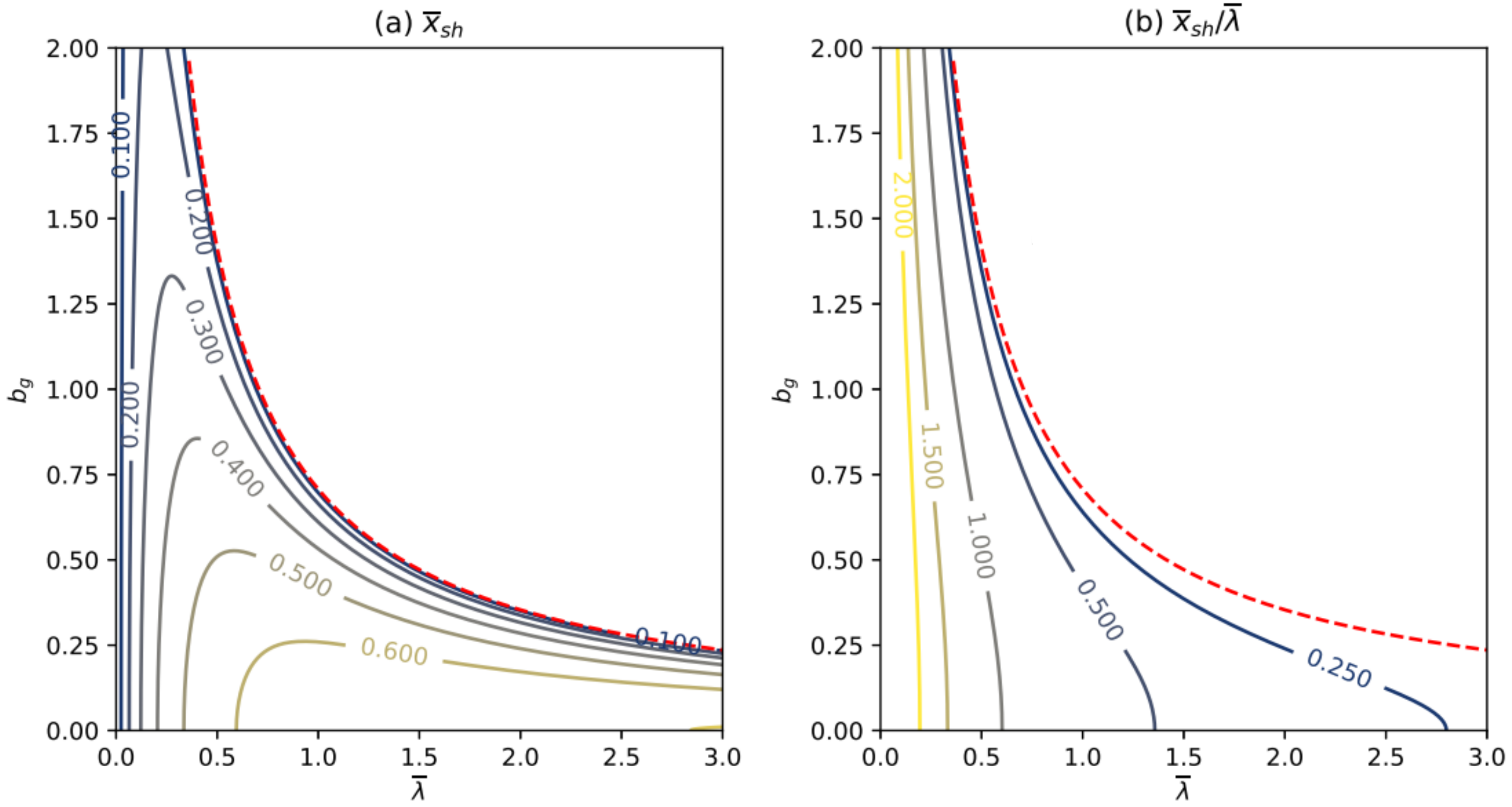


Conclusions:

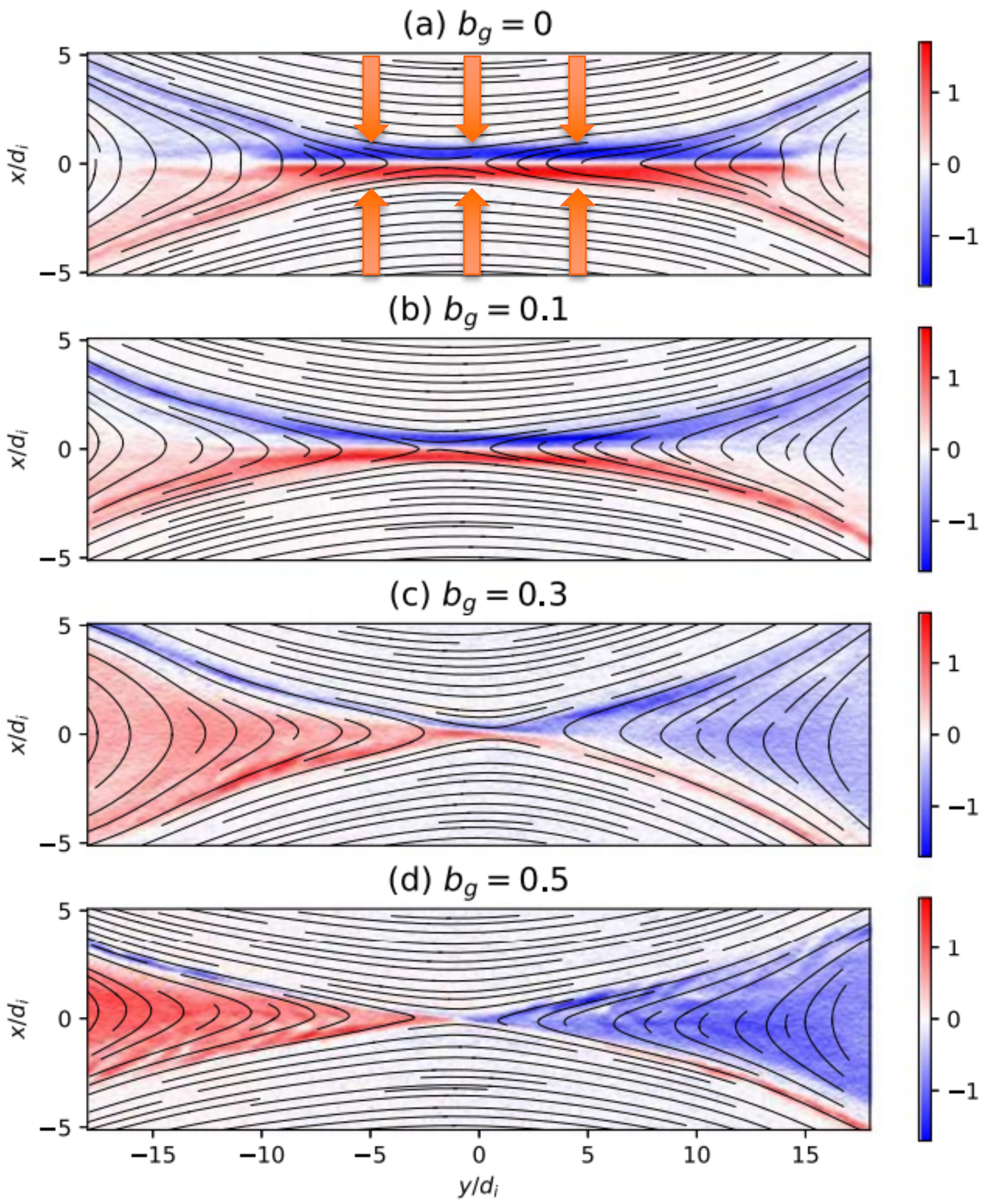
1. Heating extent ↓ as
   guide field ↑
   sheet thickness ↑

2. Above a threshold guide field, **NO** stochastic heating

Particle-in-cell (PIC) Simulation

SMILEI code
(Derouillat et al., 2018)

$$m_i/m_e = 100$$



- Black lines: **B**
- Color (arrows): E_x

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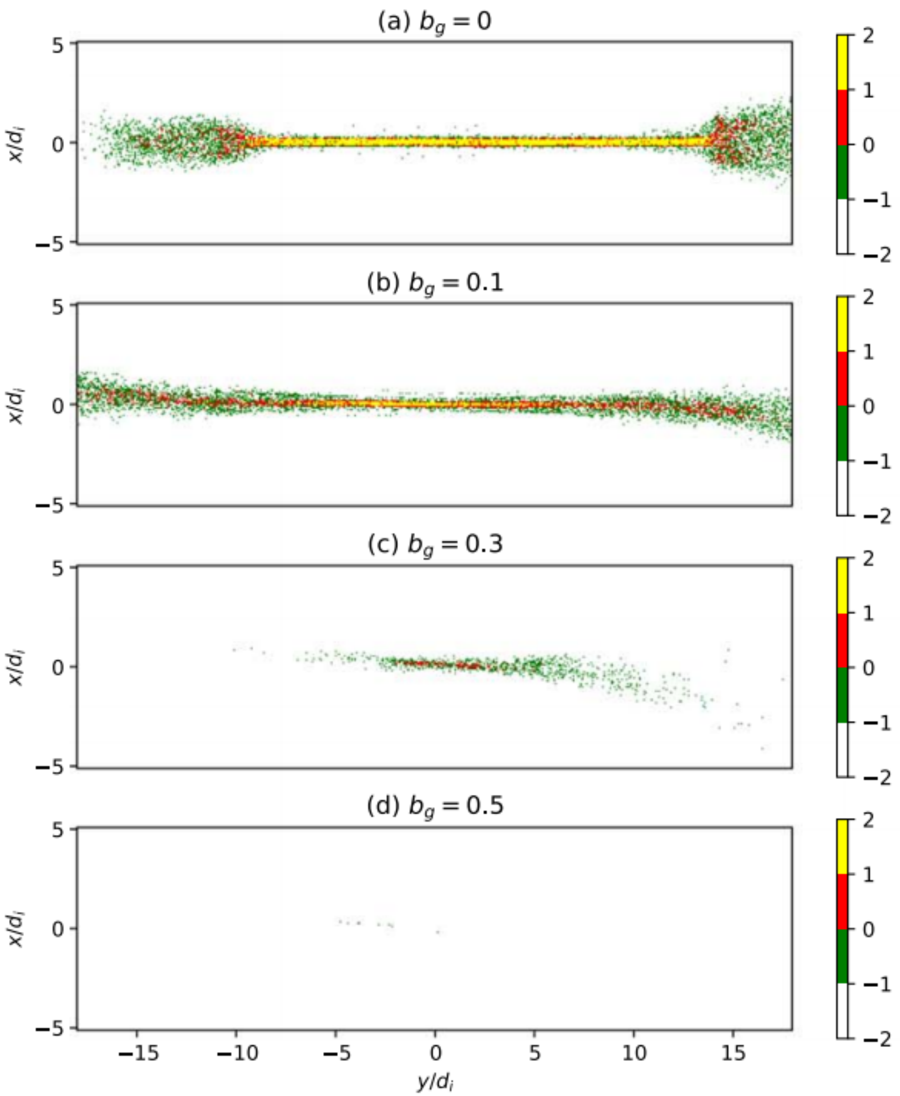
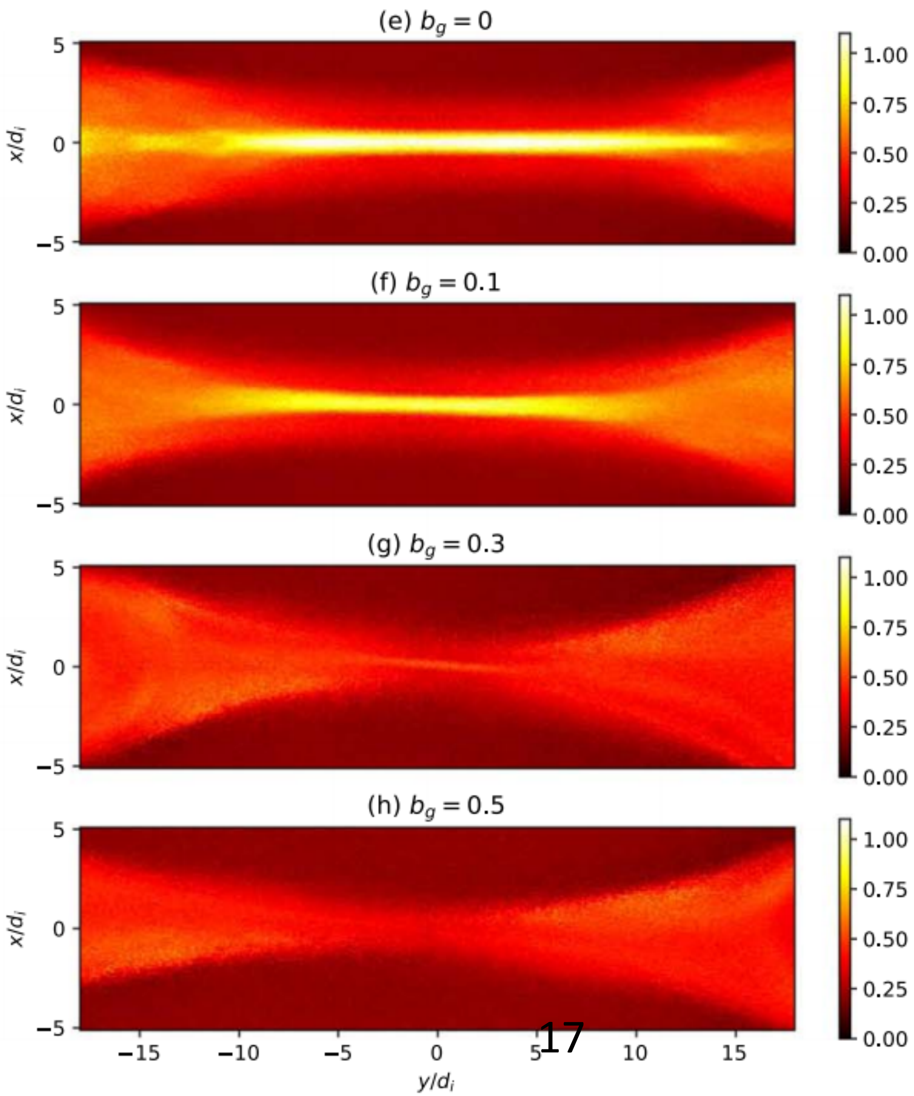
Particle-in-cell (PIC) Simulation

- Stochastic Heating Criterion



- Ion Temperature



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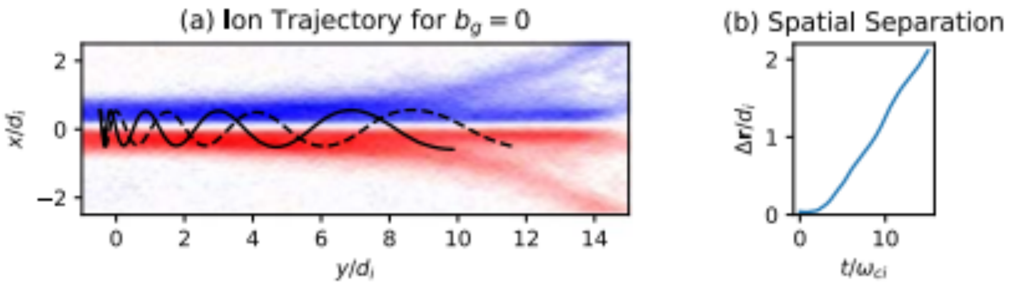
Test-Particle Simulation

- Two indicators of chaotic behavior:

  1. Spatial divergence of initially close particles

  2. Violation of magnetic moment μ

Test-Particle Simulation



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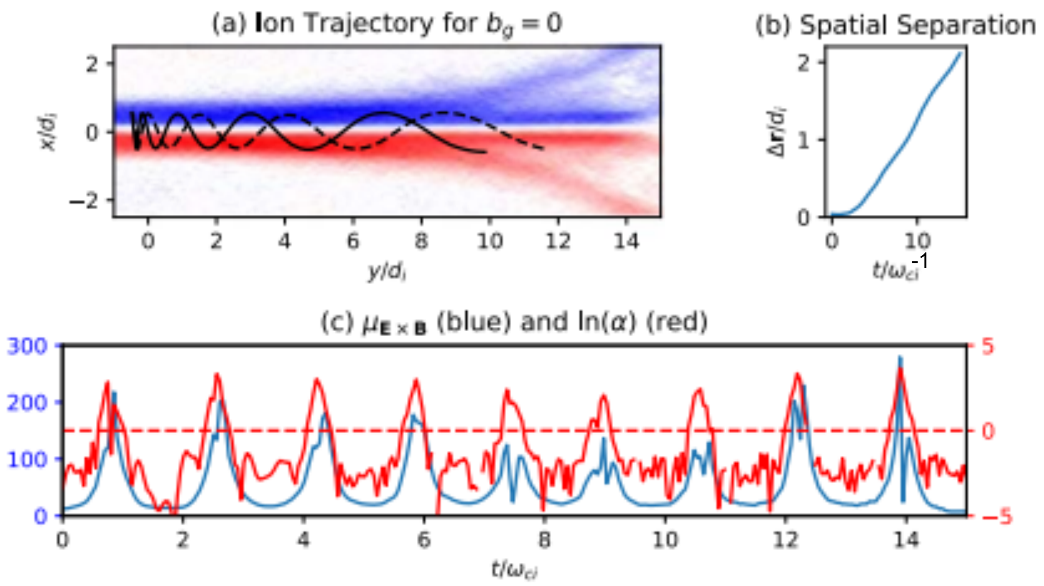

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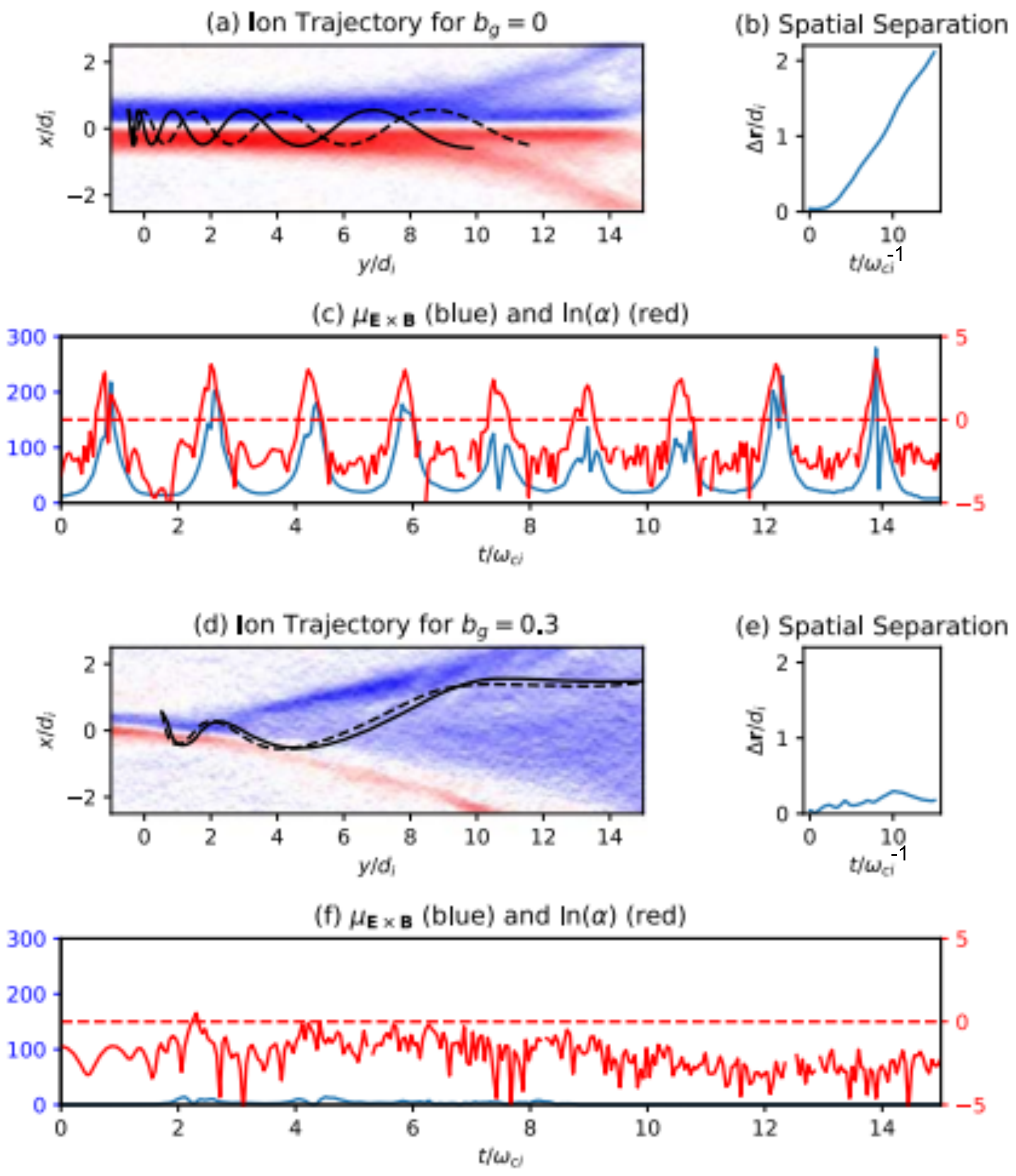

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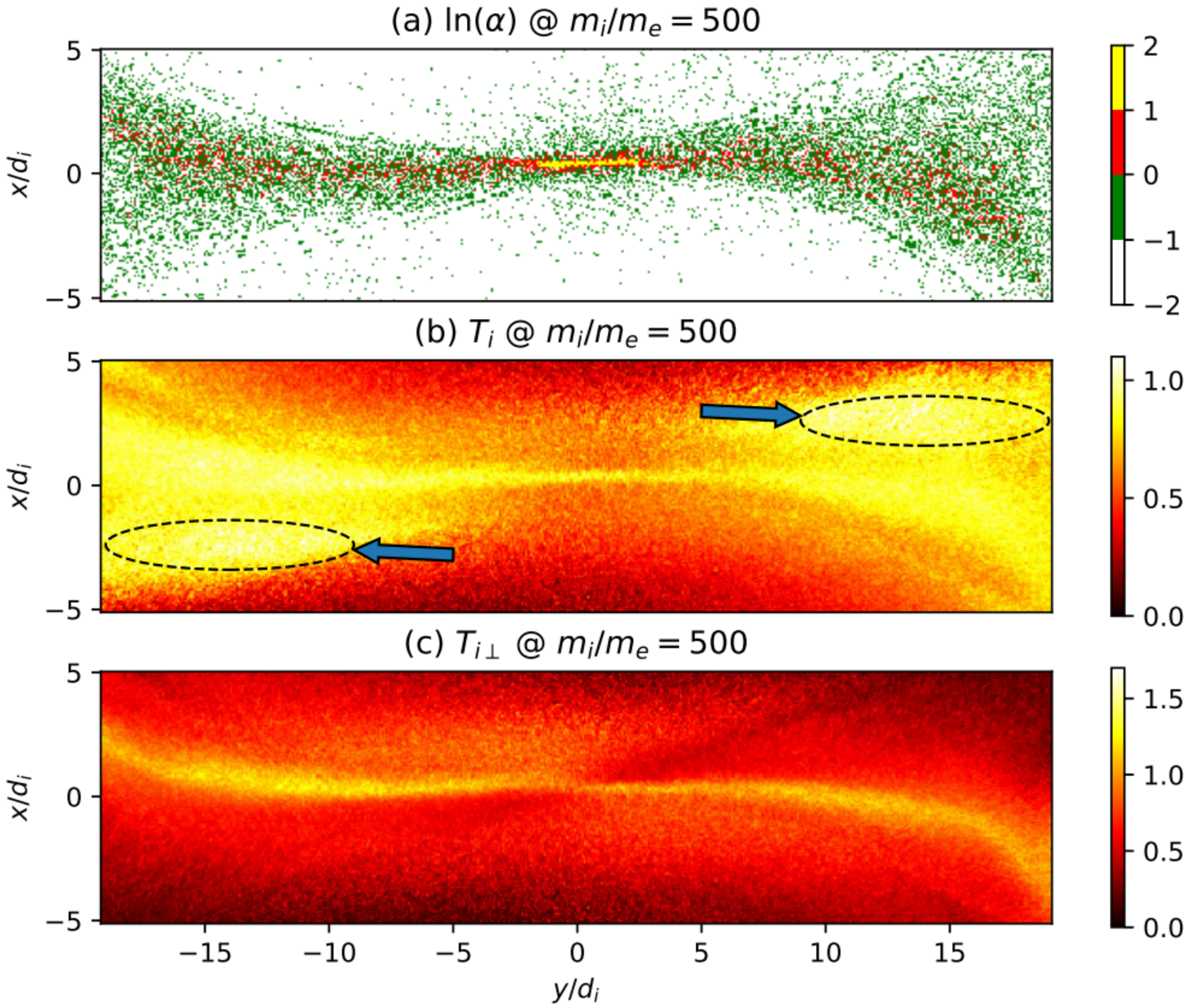
Heavy Ions

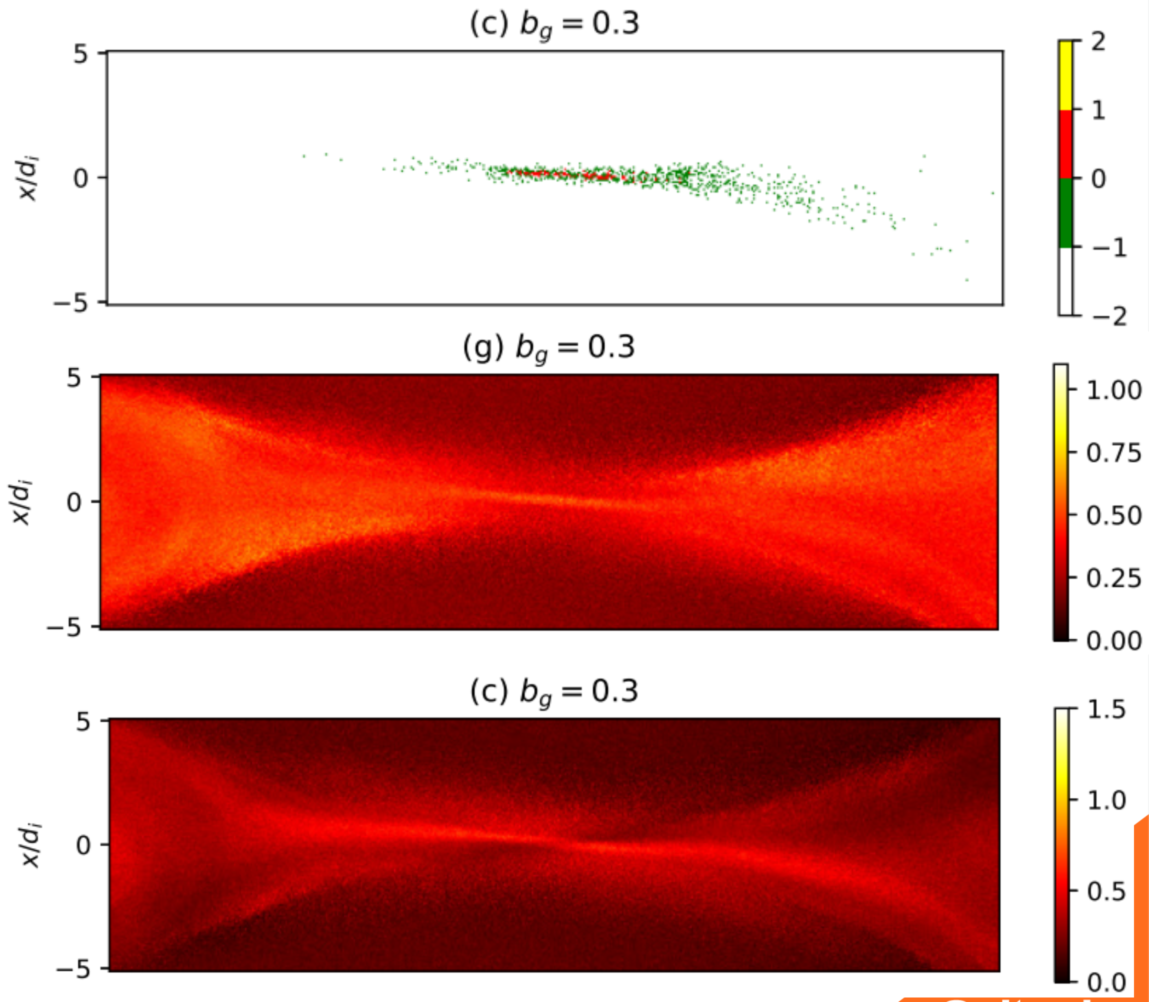
Heavy ions satisfy the criterion more easily!

$$\frac{m_i}{q_i B^2}\left|\nabla_\perp^2 \phi\right| > 1$$

$m_i/m_e = 500$



$m_i/m_e = 100$



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Conclusion

- Stochastic ion heating is intrinsic to collisionless magnetic reconnection up to moderate guide fields
- In-plane Hall electric fields render ion motions chaotic and heat the ions
- Supported by fluid analysis, kinetic analysis, and particle-in-cell simulations

Thank you
Questions?

Y. D. Yoon and P. M. Bellan, ApJL, 868, L31 (2018)

Y. D. Yoon and P. M. Bellan, ApJL, 887, L29 (2019)